Name Hania Khatagy, ID 14251

Quiz I (UTR) MTH 213, Spring 2019

Ayman Badawi

15/15

QUESTION 1. Consider the following code

k*k*k*k*k*k

For $k = 4$ *to* $(2n^4 + 5n + 3)$ → $2n^4+5n+3-4+1 = \boxed{2n^4+5n}$

$L = k^6 - 8 * k^3 + 2 * K^2 + 5$ → 13 times

For $i = 4$ *to* $(k + 3)$ $k+3-4+1 = \boxed{k}$

$S = 9 * i^3 + 5 * k - 7 * i^2 * k$ → 9 times

Next i

Next k

(1) Find the exact number of multiplication, addition and subtraction that the code will excute.

8/8

Outerloop = $13(2n^4+5n)$

$k = 4$ $\quad$ $k = 2n^4+5n+3$

$\Rightarrow (4)$ $\quad$ $\Rightarrow (2n^4+5n+3)$

innerloop $(4)(9) =$ $\quad$ $= (2n^4+5n+3)(9)$

First term = 36 $\quad$ last term = $9(2n^4+5n+3)$

$$= \left(\frac{36 + 9(2n^4+5n+3)}{2}\right)(2n^4+5n) + 13(2n^4+5n)$$

1/1

(2) Find the complexity of the code.

$O(\text{code}) = n^8$

$2\ \frac{4 \times 3}{5 \times 3} = 2\frac{12}{15}$? remainder

42

QUESTION 2. (1) Solve $6x = 12$ over planet Z_{15}.

4/4

$6x \bmod 15 = 12$

$a = 6, b = 12, n = 15$

gcd(6, 15) = 3 → # of solutions $\quad = \{2, 7, 12\}$

Is $3 | 12$? Yes

$n = gcd(a,n) \cdot d$

$15 = 3 \cdot d$

$\frac{15}{3} = d = \frac{n}{gcd(a,n)}$

$d = 5$

$x_1 = 2$

$x_2 = 7$

$x_3 = 12$

2/2

(2) Solve over planet Z $6x \ (mod\ 15) = 12$

$\Rightarrow \{2 + 5k \quad , k \in Z\}$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

Name: Hicham [illegible], ID [illegible]

Quiz II(MW) MTH 213, Spring 2019

Ayman Badawi

$\frac{15}{15}$

QUESTION 1. a) Solve $3x = 6$ over planet Z_9

$\frac{5}{5}$

$gcd(3, 9) = 3 \mid 6$

$3(x) = 6 \Rightarrow x = 2$

over planet $Z_9 \Rightarrow \boxed{x\{2, 5, 8\}}$

$9 = gcd(3, 9) \times d \Rightarrow$

$9 = 3 \times 3$

$d = 3$

b) Now solve over planet Z, $3x \ (mod\ 9) = 6$ (i.e., $3x = 6\ (mod\ 9)$)

$\frac{1}{1}$

Over planet $Z_9 \Rightarrow 3x = 6 \Rightarrow$ with $x = 2$.

thus in $\boxed{2+3k}$ with $k \in Z$ over planet Z.

QUESTION 2. Consider the system

$$x = 7\ (mod\ 10)$$
$$x = 7\ (mod\ 11)$$

a) Find the unique solution over Planet Z_{110}

$\frac{8}{8}$

$gcd(10, 11) = 1$ CRT applies

$n = 10 \cdot 11 = 110$

$m_1 = \frac{n}{n_1} = \frac{110}{10} = 11$

$m_2 = \frac{n}{n_2} = \frac{110}{11} = 10$

m_1^{-1} in Z_{10}

$11x = 1 \Rightarrow x = 1$ in Z_{10} $(m_1^{-1} = 1)$

m_2^{-1} in Z_{11}

$10x = 1 \Rightarrow x = 10$ in Z_{11} $(m_2^{-1} = 10)$

Then $x = a_1 m_1 m_1^{-1} + a_2 m_2 m_2^{-1} = (7 \times 11 \times 1) + (7 \times 10 \times 10) = 777 \bmod 110 = 7$.

$x = 7$ is a unique solution over Z_{110}

b) Find all solutions over PLANET Z.

$\frac{1}{1}$

Over planet $Z \Rightarrow \boxed{7 + 110k}$ with $k \in Z$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

Quiz II(MW) MTH 213, Spring 2019

Ayman Badawi

15/15

QUESTION 1. a) Solve $3x = 6$ over planet Z_9

a, b, n

① check if solution exists

$\gcd(3,9) = 3$ and $3 \mid 6$ ∴ Yes, and there are three solutions

5/5

② $x_1 = 5$ (by trial & error)

③ compute difference, d ★ $n = \gcd(a,n)\, d$

$9 = 3d \longrightarrow d = 3$

④ $x_2 = 5+3 = 8$; $x_3 = 8+3 = 11$ ★ mod 9 $\Rightarrow x_3 = 2$

∴ $\left\{ \begin{array}{l} x_1 = 5 \\ x_2 = 8 \\ x_3 = 2 \end{array} \right\}$ over Planet Z_9

b) Now solve over planet Z, $3x \ (mod\ 9) = 6$ (i.e., $3x = 6\ (mod\ 9)$)

1/1

general solution over planet Z:

$\boxed{2 + 3k,\ k \in Z}$

QUESTION 2. Consider the system

$$x = 7\ (mod\ 10)$$
$$x = 7\ (mod\ 11)$$

a_1, n_1, a_2, n_2

a) Find the unique solution over Planet Z_{110}

8/8

① check if CRT applies

$\gcd(10, 11) = 1$ ∴ Yes, CRT applies

② $n = n_1 n_2$

$= 10 \cdot 11 = 110$

∴ there is a unique solution over Z_{110} (given)

③ $m_1 = \frac{n}{n_1} = \frac{11 \cdot 10}{10} = 11$ $m_2 = \frac{n}{n_2} = 10$

④ m_1^{-1} in Planet Z_{n_1} m_2^{-1} in Planet Z_{n_2}

$11 = 1$ in Z_{10} → $(11)^{-1}$ in Z_{10} $(10)^{-1}$ in Z_{11}

$\boxed{11x = 1} \rightarrow x = 1$ $\boxed{10x = 1} \rightarrow x = 10$

★ unique soln., $x = a_1 m_1 m_1^{-1} + a_2 m_2 m_2^{-1}$

$= 7 \cdot 11 \cdot 1 + 7 \cdot 10 \cdot 10 = 777 = 7$ in Planet Z_{110}

b) Find all solutions over PLANET Z.

1/1

general solution over Planet Z:

$\boxed{7 + 110k,\ k \in Z}$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

Name ~~~~ ______, ID ______

Quiz IV (UTR) MTH 213, Spring 2019

Ayman Badawi

QUESTION 1. Let $A = \{3, -2, \{7\}, 12, 23\}$. Write Down T or F

$P(A): \{A, \emptyset, \{3\}, \{-2\}, \{\{$

(i) $\{3, 12\} \in P(A)$ T

(ii) $\{7\} \in P(A)$ F

(iii) $\{\{7\}, 23\} \subseteq P(A)$ F

(iv) $\{7\} \in A$ T

(v) $|P(A)| = 10$ F

$\frac{15}{15}$

QUESTION 2. Let $W = \{-4, -3, 2, 5, 7, 0, 23, 42\}$ (be the universal set). Given $A = \{2, 7, 23, 42\}$ and $B = \{5, 23, 42\}$.

(i) Find $A - B$.

$A - B: \{2, 7\}$

(ii) Find $B - A$.

$B - A: \{5\}$

(iii) Find $A \cap B$

$A \cap B: \{23, 42\}$

(iv) Find $A \cup B$

$A \cup B \{2, 5, 7, 23, 42\}$

(v) Find $\overline{B}$

$\overline{B}: \{-4, -3, 2, 7, 0\}$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

Abdelaziz Mostafa
b00074028

Quiz IV (MW) MTH 213, Spring 2019

Ayman Badawi

$\frac{15}{15}$

QUESTION 1. a)Let $m = 300$. How many $0 < integers < m$ are there where the $gcd(integer, 300) = 4$

Let $n = 4$.

Then: $\frac{m}{n} = 75 = 5 \cdot 15 = 5 \cdot 5 \cdot 3$

Hence $\varphi(\frac{m}{n}) = (5-1) \cdot 5^1 \cdot (3-1) = 40$.

~~b) Find $5^{14} \pmod 7$~~ There are 40 integers between 0 and m s.t. $gcd(\text{any } \varphi(\frac{m}{n}), 300) = 4$.

$\frac{5}{5}$

QUESTION 2. Let $W = \{2, 5, 7, 0, 23\}$ (be the universal set). Given $A = \{2, 0, 23\}$ and $B = \{5, 23\}$.

(i) Find $A - B$.

$A - B = \{2, 0, 23\} - \{5, 23\} = \{2, 0\}$

(ii) Find $B - A$.

$B - A = \{5, 23\} - \{2, 0, 23\} = \{5\}$

$\frac{10}{10}$

(iii) Find $A \cap B$

$A \cap B = \{23\}$

(iv) Find $A \cup B$

$A \cup B = \{0, 2, 5, 23\}$

(v) Find $\overline{B}$

$\overline{B} = \{0, 2, 7\}$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

Quiz V (MW) MTH 213, Spring 2019

Ayman Badawi

15/15

QUESTION 1. Find 5^{51} $(mod\ 36)$

$gcd(36, 5) = 1 \Rightarrow \phi(n) = (2-1)\cdot 2\cdot(3-1)\cdot 3$

$36 = 2^2 \cdot 3^2$

$\phi(n) = 12$

$51 = \phi(n)\,\boxed{\ }\, + r$

$51 = 12\,\boxed{4} + 3$

$5^{12\cdot 4} \cdot 5^3 \pmod{36} = 5^3 \pmod{36} = 17 \pmod{36}$

$= 17$

QUESTION 2. Let $M = \{\{2\}, 2, 5, 7, 0, \{23\}\}$. Write down T or F.

$|M| = n(M) = 6$

$|P(M)| = n(P(M)) = 2^6 = 64$

(i) $\{2\} \in M$ T

(ii) $\{7, \{23\}\} \in P(M)$ T

(iii) $\{\{2\}, \{5, 7\}\} \subseteq P(M)$ T

(iv) $|P(M)| = 12$ F

(v) $\{7, \{2\}\} \subseteq P(M)$. F

QUESTION 3. Given $a_n = 2a_{n-1} + 8a_{n-2}$, where $a_0 = 8$ and $a_1 = 14$. Find an equation that describe the sequence a_n.

$$\frac{x^n}{x^{n-2}} = \frac{2x^{n-1}}{x^{n-2}} + \frac{8x^{n-2}}{x^{n-2}}$$

$x^2 = 2x + 8 \Rightarrow x^2 - 2x - 8 = 0$

$\left.\begin{array}{l} x = 4 \\ x = -2 \end{array}\right\}$ 2 distinct solutions

$a_n = c_1(4^n) + c_2(-2^n)$

$\left.\begin{array}{l} a_0 = 8 = c_1 + c_2 \\ a_1 = 14 = 4c_1 - 2c_2 \end{array}\right\}$ $c_1 = 5$, $c_2 = 3$

$a_n = 5(4^n) + 3(-2^n)$

Find a_7 $a_7 = 5(4^7) + 3(-2^7) = 81536$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

Ismat Maarouf
74072

Quiz 6 (UTR) MTH 213, Spring 2019

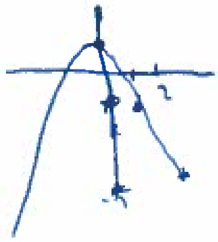
Ayman Badawi

25/25

QUESTION 1. Consider the following

1) $F : (-\infty, 2] \to (-\infty, -8]$, s.t. $f(x) = -2x^2 + 1$. If F is not a function, then change the codomain so that it becomes a function but not ONTO

$f(x) = -2x^2+1$



$\Rightarrow$ co-domain $= (-\infty, 2]$

$F : (-\infty, 2] \to (-\infty, 2]$

2) $f : (0, 3] \to [0, 9)$ s.t $f(x) = -x^2 + 9$. Then $f(x)$ is a function. Is F bijective? if yes then find f^{-1}. If not, then give me a reason.

Range $= [0, 9) =$ co-domain
$\Rightarrow$ onto $\Rightarrow$ Surjective
& 1-1 $\Rightarrow$ Bijective

$f^{-1} : [0, 9) \to (0, 3]$ $\quad y = -x^2+9 \Rightarrow x = -y^2+9 \Rightarrow y^2 = 9 - x \Rightarrow \boxed{y = \sqrt{9-x}}$

3) Given $f : [4, 12] \to [2, 4]$ is a bijective function such that $f(x) = \sqrt{2x+1} - 1$. Find the domain and the range of f^{-1}. Then find a formula for f^{-1}.

$f^{-1} : \underbrace{[2, 4]}_{\text{Domain}} \to [4, 12]$ codomain

$y = \sqrt{2x+1} - 1 \Rightarrow x = \sqrt{2y+1} - 1$
$\Rightarrow \sqrt{2y+1} = x + 1$
$\Rightarrow 2y + 1 = (x+1)^2$
$\Rightarrow 2y = (x+1)^2 - 1$
$\Rightarrow \boxed{y = \frac{(x+1)^2 - 1}{2}}$

4) Let $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 5 & 3 & 6 & 2 & 7 & 9 & 8 & 1 & 10 & 4 \end{pmatrix}$

a) Find F^2

$f^2 = f \circ f \Rightarrow f^2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 7 & 6 & 9 & 3 & 8 & 10 & 1 & 5 & 4 & 2 \end{pmatrix}$

b) Find the least positive integer $n \geq 1$ such that $f^n = I$.

Rewrite: $f = \underbrace{(1\ 5\ 7\ 8)}_{4\text{-cycle}} \circ \underbrace{(2\ 3\ 6\ 9\ 10\ 4)}_{6\text{-cycle}}$

$\Rightarrow \text{LCM}(4, 6) = \frac{24}{2} = 12 = n \Rightarrow f^n = I, \boxed{n = 12}$

$\Rightarrow f^{12} = I$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

Quiz 6 (UTR) MTH 213, Spring 2019

Ayman Badawi

Hania khafagy
74231

~~14~~ 15/15

QUESTION 1. Consider the following

1) $F : (-\infty, 2] \to (-\infty, -8]$, s.t. $f(x) = -2x^2 + 1$. If F is not a function, then change the codomain so that it becomes a function but not ONTO

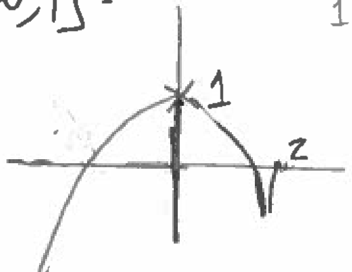
3

Range $= (-\infty, 1]$.

Hence codomain ~~Range~~ $= (-\infty, b), b > 1$

onto = codomain = range

1 - 1



$y = -2x^2 + 1$

$x = 0 \quad y = 1$

$y = 0 \quad x = -\sqrt{\frac{1}{2}}$

$\frac{1}{2} = x^2$

range $= (-\infty, 1)$

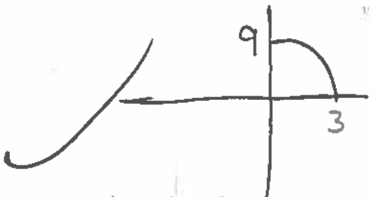
note $f: (-\infty, 2] \to \mathbb{R}$ is ok too

$F: (-\infty, 2] \to (-\infty, 2]$

range $\neq$ codomain

2) $f : (0, 3] \to [0, 9)$ s.t $f(x) = -x^2 + 9$. Then $f(x)$ is a function. Is F bijective? if yes then find f^{-1}. If not, then give me a reason.

3



range $= (0, 9]$

range = codomain onto ✓

$y = 0$

$x = \sqrt{9} = \pm 3$

1 - 1 ✓

onto (codomain = range)

1 - 1 Yes, it's bijective

$f^{-1}(x) =$

$x = -y^2 + 9$

$y = \sqrt{9 - x}$

$f^{-1}(x) = \sqrt{9 - x}$

3) Given $f : [4, 12] \to [2, 4]$ is a bijective function such that $f(x) = \sqrt{2x + 1} - 1$. Find the domain and the range of f^{-1}. Then find a formula for f^{-1}.

3/3

Domain codomain

$f. [4, 12] \to [2, 4]$

bijective

ONTO ✓

1 - 1 ✓

~~Domain of f^{-1}~~

~~codomain of f^{-1}~~

$f(x) = \sqrt{2x + 1} - 1$

$f^{-1}: [2, 4] \to [4, 12]$

$y = \sqrt{2x + 1} - 1$

$x = \sqrt{2y + 1} - 1$

$\frac{(x + 1)^2 - 1}{2} = y$

$f^{-1}(x) =$

$f^{-1}(x) = \frac{(x + 1)^2}{2} - 1$

4) Let $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 5 & 3 & 6 & 2 & 7 & 9 & 8 & 1 & 10 & 4 \end{pmatrix}$

2

a) Find F^2

$f^2 = (f \circ f) = f(f(1)) = f(5) = 7$

$f^2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 7 & 6 & 9 & 3 & 8 & 10 & 1 & 5 & 4 & 2 \end{pmatrix}$

4

b) Find the least positive integer $n \geq 1$ such that $f^n = I$.

$(\underbrace{1\ 5\ 7\ 8}_{4\text{cycle}})(\underbrace{2\ 3\ 4\ 6\ 9\ 10}_{6\text{cycle}})$

$\text{LCM} = \frac{(4 \times 6)}{\gcd(4, 6)} = \frac{4 \times 6}{2} = \frac{24}{2} = 12$

$F^{12} = I$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

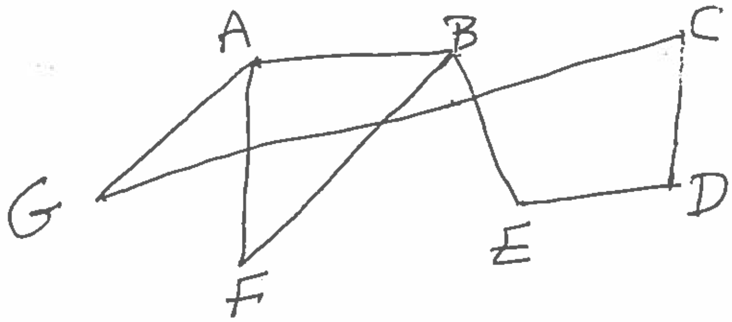
Quiz 7 (MW) MTH 213, Fall 2018

Natasha M.
76735

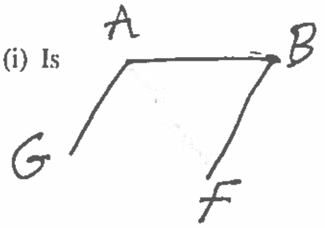
Ayman Badawi

15/15

QUESTION 1. Consider the following Graph G.



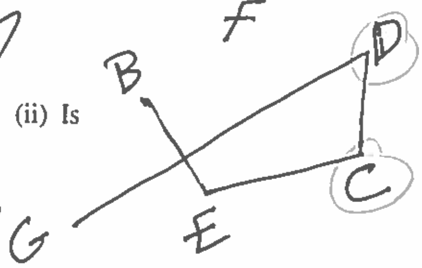
(i) Is



an induced subgraph of G? Explain No.
Because the edge A–F exists in G but does not show in this graph.

3/3

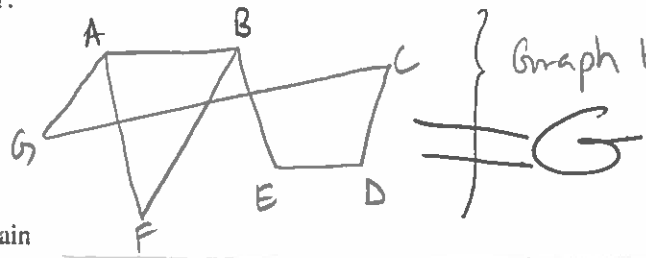
(ii) Is



a subgraph of G? explain No, ~~bc~~ edges G–D and E–C
~~Yes, because~~ ~~$E_H \subseteq E_G$~~ do not exist in E_G
~~and~~ ~~$V_H \subseteq V_G$~~ $\therefore E_H \not\subseteq E_G$

2/2

(iii) If H is an induced spanning sub-graph of G, then draw H.
induced AND spanning only when $H = G$ $\therefore$
requires that $V_H = V_G$



3/3

(iv) Is A–B–E–B–F a path in G? explain
No, because a path is $V_i - V_{i_1} - V_{i_2} - \cdots - V_{i_n}$ where $V_i, V_{i_1}, V_{i_2}, \ldots, V_{i_n}$ are distinct vertices.
but here B is repeated, thus, it is not a path.

2/2

(v) Is D–E–B–F–A–G–C–D a cycle in G? explain
Yes, because by definition, a cycle is $V_i - V_{i_1} - V_{i_2} - \cdots - V_{i_n} - V_i$ where $V_i, V_{i_1}, V_{i_2}, \ldots, V_{i_n}$ are distinct vertices.

2/2

What does it mean when you say this graph is connected?
for every two vertices in the graph ∃ a path between them, connecting them.
there exists

3/3

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com